White Discrimination in Provision of Black Education: Plantations and Towns

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Abstract

We present a model of public provision of education for blacks in two discriminatory regimes, white plantation controlled, and white town controlled. We show that the ability to migrate to non discriminating district constrains the ability of both types of whites to discriminate. The model produces time series of educational outcomes for whites and blacks that mimic the behavior seen in Post Reconstruction South Carolina to the onset of the Civil Rights Act.

This paper examines the evolution of human capital, specifically education, of blacks and whites under discrimination. From the end of the US Civil War to the presidential election of 1876, former states in the Confederacy were under black majority rule. Table 1 presents the time series evidence on the population of South Carolina from 1790 to the present as well as the racial composition of the population.

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Table 2 provides similar information for other southern states, Alabama, Georgia, Louisiana, Mississippi, North Carolina and Virginia.[1] For the censuses of 1820 to 1920, inclusive, blacks were the majority population in South Carolina. The pattern of rising black share in southern states from founding to the 1880 census is common for all states in Table 2 except for Virginia. Prior to the Civil War the overwhelming majority of blacks in the southern states of the US were slaves. Table 2 also presents the share of blacks in the US population that live in the 11 southern states that formed the Confederate States of the America during the Civil War.[2] Notice that the share of blacks living in these southern states fluctuated from 73 percent to 83 percent between 1790 and 1920. However from 1920 onward there was a sharp decrease in the share of blacks that resided in these southern states. After peaking in 1880 at 83 percent, the share of blacks residing in the south declined to 77 percent in 1920 and has fallen continuously (except for the 1990-2000 period). The contested 1876 presidential election was determined by an Election Commission set up by the US Congress. The election commission, by partisan vote, determined that the contested results in South Carolina, Louisiana and Florida were won by Hayes, the Republican candidate. Hayes announced that he would remove federal troops in the southern states, effectively ending Reconstruction. With the removal of federal troops in the southern states, these states rewrote their constitutions in order to disenfranchise blacks as well as poor whites. By the 1890s, most southern states reverted to white rule and discrimination against blacks became standard policy. Despite this discriminatory regime against blacks, black education measures still converged to the education measures of their

[1] The data for 1790-1970 come from *Historical Statistics of the United States: Colonial Times to 1970*, 1980 data come from *Datapedia of the United States: 1790-2005*, 1990 and 2000 data come from *Statistical Abstract of the United States, 1992* and *2002.*

[2] South is defined as the 11 states that formed the old Confederate States of America: Alabama, Arkansas, Florida, Georgia, Louisiana, Mississippi, North Carolina, South Carolina, Tennessee, Texas and Virginia.

white counterparts. In particular education inputs such as class size, school year length and teacher salary for blacks became much more similar to whites throughout the period 1880-1960, prior to the Civil Rights Acts of the 1960s, as well as prior to the 1954 Brown vs. Board of Education.[3]

We develop a model to describe two separate discriminatory regimes, a plantation system and a yeoman system. In the plantation system, whites are monopsony employers of black sharecroppers as well as monopoly providers of education to the children of sharecroppers in addition to their own children. In the yeoman system whites determine the tax rates to finance education both for their children and the children of black yeoman. Unlike the plantation system, white and black yeoman produce output in a nondiscriminating market. What constrains both sets of whites is the ability of black sharecroppers and black yeoman to migrate to a completely nondiscriminatory neighborhood where they can work without discrimination and provide for their own children's education. However migration to this nondiscriminating neighborhood is costly. We solve for the perfect foresight equilibrium in both districts, where discriminating whites choose the level of discrimination against blacks in order to maximize white utility subject to the migration constraint.[4]

Using county level data for South Carlina, we are able to calculate the average class size for whites and blacks, average expenditures per pupil for whites and blacks and average expenditures per teachers for whites and blacks from 1880-1964.[5] After 1964 the information on education no longer comes broken down by race. However

[3] For school year length, see Canaday (2003) for details of black convergence.

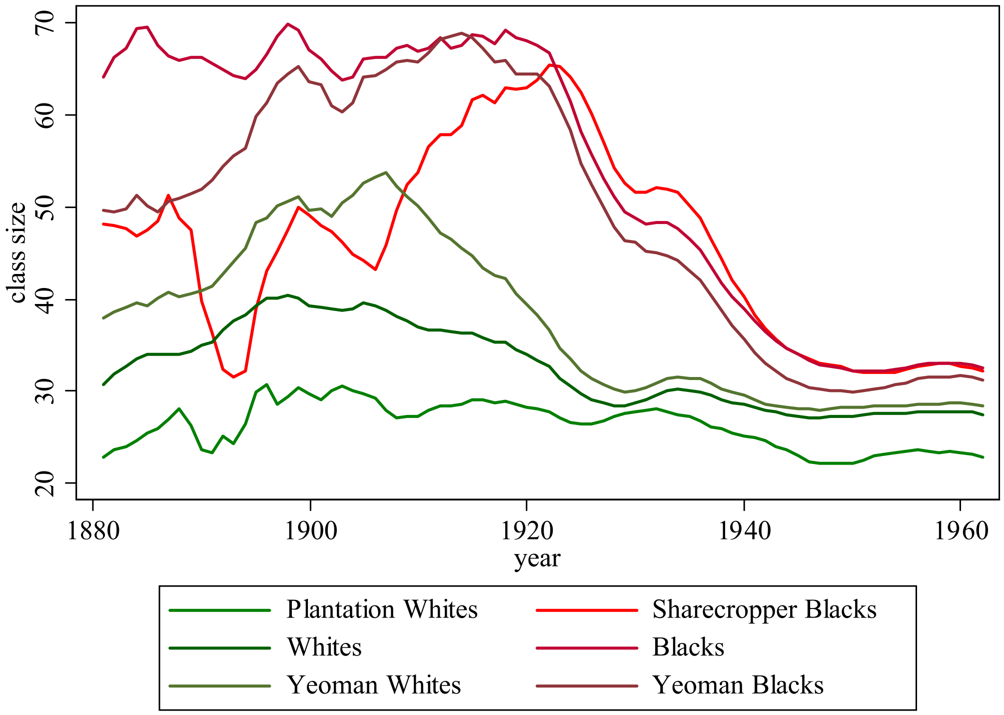
[4] The focus of this paper is the convergence of black education with white education in South Carolina during Jim Crow, 1880-1966. However there is evidence that the south lagged the rest of the country in educational attainment from 1840-2000, so convergence by South Carolina blacks to South Carolina white education levels would still entail a gap between them and the rest of the US. For regional differences see Baier, et al. (2004).

[5] For more on the data see Canaday (2003).

what the data do document is that black class sizes, spending per black pupil and spending per black teacher all converge to the levels of these variables of whites. This holds for blacks in overwhelmingly black counties that hereafter we refer to as plantation counties, as for blacks in white counties, hereafter referred to as yeoman counties.[6] Figure 1 illustrates this point for class size.[7] The figure also contains the average class size for whites and blacks at the entire state level in South Carolina. Observe that average class size has declined for whites from roughly 1900 to 1964, with a noticeable exception for the Great Depression. There is a large disparity, however, between whites prior to the 1930s. Yeoman whites had steadily increasing class size from 1880 to about 1910, while plantation whites show a much more muted rise. Class size reductions become noticeable from 1920 to the end of the period for both blacks at the statewide level and yeoman blacks, and with a slight delay for sharecropper blacks. Average class size for blacks is essentially constant from 1880 to 1920 at the state level, rising for yeoman blacks and widely varying for sharecropper blacks.

[6] When defining county type we followed a rule that generally high black population counties were plantation counties. However there are exceptions to this rule; for example Beaufort county had a high black population share early on but looked more like a yeoman county due to land confiscation by troops during the Civil War.

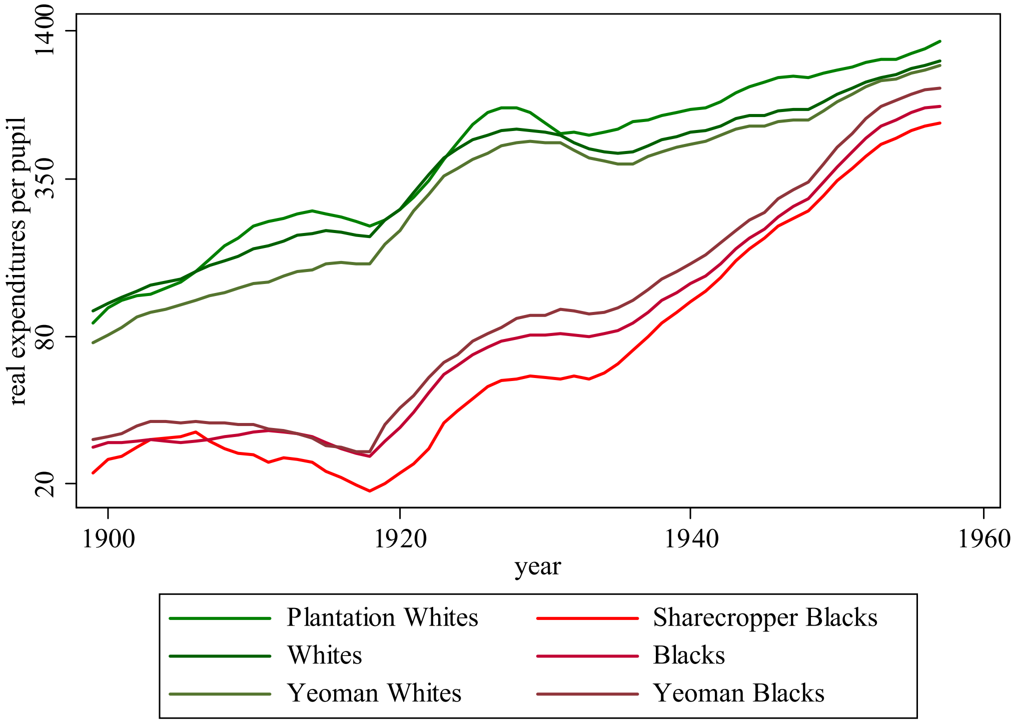
[7] Plantation whites covers white children in the counties of Allendale, Calhoun, Dorchester, Hampton and Jasper. These counties account for sharecropper blacks. Yeoman whites and blacks are those in Anderson, Cherokee, Greenville, Oconee, Pickens, Spartanburg and York counties.



Average class size for blacks and whites

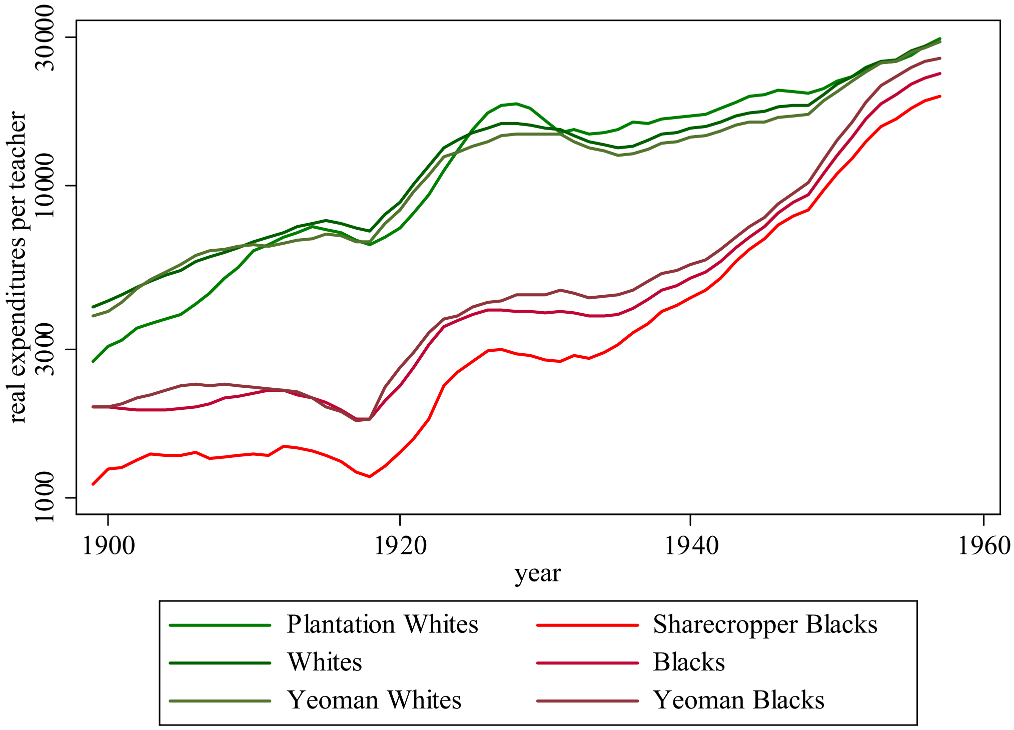
Figure 2 presents information on real spending per pupil for whites and blacks from 1900 to 1962. We present the information in constant 2000 dollars. These expenditures do not include capital expenditures. There are two distinct periods in the figure. From 1897-1920 real spending per black pupil was constant at the state level and for yeoman blacks. However for sharecropper blacks, real spending per pupil falls from roughly 1905 to the end of World War I. At the conclusion of World War I, real spending per black pupil rises rapidly, stagnating during the Great Depression, and rising from about 1936 until the end of the sample. For whites there is mostly a steady rise in spending per pupil. The 1920s was a period of rapid increases, but spending in 1960 is about where it would have been expected based on a simple trend from 1897 to 1959. Notice that the gap between whites and blacks are substantially reduced by the end of the period. In 1897 white spending per pupil was $88, whereas black spending per pupil was $27, or 31 percent. By 1959 white spending per pupil had reached

$1128 while black spending per pupil was $734, or 65 percent. The rate of growth of spending was 4.2 percent per year for whites, and 5.5 percent per year for blacks.



Real expenditures per pupil, 2000 dollars.

Finally we present expenditures per teacher. We chose this measure instead of teacher salaries in order to measure resources available to teachers as opposed to salaries. Figure 3 presents the data for the period 1897 to 1959.



Real expenditures per teacher, 2000 dollars.

The same pattern emerges as with real spending per pupil, however because class size reductions were greater for blacks than for whites, there is slower convergence in real spending per teacher than in real spending per pupil. As shown in Tamura (2001) and in the model here, this is crucial for the story of convergence during the period of discrimination. In 1897 real spending per teacher for whites and blacks were \$3570 and \$1849, respectively. In 1959, the last year data are available, spending per teacher for whites and blacks were \$31750 and \$24,445, respectively. Black spending per teacher converged from 52 percent of white spending per teacher in 1897 to 77 percent of white spending per teacher in 1959. The rates of growth of spending per white teacher and black teacher respectively are 3.6 percent and 4.3 percent.

The next section provides a brief overview of the literature. In the third section of the paper we present a model of discrimination by whites against blacks. We focus on three different types of work-school districts. The first is a plantation district where plantation whites combine land, their human capital and the total human capital of their black sharecroppers to produce output. In addition to their monopsony hiring power, plantation whites are the monopoly providers of education for sharecropper blacks. The second district is a yeoman district where whites and blacks produce output in competitive input and output markets, but whites are the median voter as well as the school superintendent in the school district. Finally the third district is one in which blacks can migrate to at a cost in which they can produce output in competitive input and output markets, and also they are the median voter in the school district. The fourth section produces a comparison of time paths of human capital, class size and average spending per pupil between whites and blacks in two types of counties. One county includes plantation districts and districts where blacks face no discrimination, the other county includes yeoman districts and districts where blacks face no discrimination. The final section concludes.

PREVIOUS WORK

Recent work has been produced by leading scholars on the role of education in the south on convergence in economic status of blacks. This is a return to a fundamental question introduced by Myrdal (1948). Smith (1984,1986) and Smith and Welch (1989) examine the role of human capital accumulation by blacks in explaining economic convergence of blacks relative to whites. Smith (1984,1986) shows that the crucial periods of Reconstruction in the south, 1865-1876, and Jim Crow discrimination from 1880-1964, provide evidence on the importance of human capital for explaining black white earnings differentials. In the first period, blacks were majority voters and hence controlled public education. During this short period black human capital accumulation would be much greater than during the period after Reconstruction when blacks increasingly became disenfranchised. However for those born in 1860, Reconstruction provided a maximum 11 year period in which they could be educated practically through high school. The cohort that had this enhanced access to education, would perform differently in the labor market as adults compared to unfortunate blacks that were educated during the period of Jim Crow. Margo (1986a,b,1990) provides evidence of educational achievement in segregated schools and the effects of Jim Crow discrimination. He shows that despite discrimination, blacks did improve relative to whites, albeit more slowly than they would if education were not subject to discrimination. Smith and Welch (1989) document the convergence in educational achievement of blacks and whites throughout the 20th century. Margo and Finegan (1993) show that rising school enrollment was responsible for the decline in labor force participation of black teenagers. Butler, Heckman and Payner (1989), Heckman and Payner (1989), Heckman (1990), Heckman et al (2000) and Donohue et al. (2002) examine the importance of human capital accumulation, federal intervention and migration in explaining black economic progress in the 20th century. All of these papers focus on the importance of education on future economic performance of individuals. The papers by Heckman et al, however highlight the importance of federal intervention in improving black outcomes. Orazem (1987) is quite similar to this paper in many dimensions. He examined two outcomes from segregated schools in Maryland from 1924-1938, school daily attendance and reading skills. He found that school inputs significantly explained both variation in average daily attendance as well as reading scores. Had blacks received more equal funding, their human capital accumulation would have been substantially enhanced. Bowles (1970) provides evidence that better educated blacks were more likely to leave the south between 1955 and 1960 than poorly educated blacks.

Related to this literature is the work of Benabou (1993,1996ab). His work considers the macroeconomic implications of public schooling in a heterogeneous human capital world. Using a model with agglomeration returns to specialization, as in Tamura (1992, 1996, 2002, 2004), he shows that heterogeneity is detrimental not only for production in a static sense, but detrimental for economic growth. Differing education finance regimes produces differing rates of human capital convergence. Tamura (2001) provides a microfoundation for diminishing returns to educational resources. This is the crucial feature that produces convergence in human capital. Diminishing returns to educational resources arises when teacher quality is relatively more important for human capital accumulation than class size (or teacher mentoring or teacher discipline). In this paper we assume an extreme form of inequality in educational resources. Discriminating whites are able to extract rents from poor blacks. However in spite of this horrendous environment, black progress is merely hindered, not eliminated.

MODEL: DISCRIMINATION

In this section we present the basic model of discrimination. There are two races of individuals, blacks and whites. There are two discriminatory regimes, plantations and towns. On plantations, whites determine the consumption of their black sharecroppers, the education expenditures for black sharecropper children, their own consumption and the education expenditures for their own children. White plantation owners understand that the more consumption that they allocate to their sharecroppers, the greater proportion of sharecroppers remain. The more that they spend on the education of sharecropper children, the greater proportion of sharecroppers remain. There is a tradeoff, however in educational expenditures on sharecropper children. When these children become adults, the more human capital they have, the easier it is for them to migrate to a nondiscriminating district, but the more human capital they have, the more productive they are on the plantation. In towns, white yeomen have different decision rules. White yeoman control the tax rate on their own income, the tax rate on the black yeoman population and expenditures on their children's education and the children of black yeoman. Unlike white plantation owners, white yeoman do not employ black yeomen, their return from discrimination is diverting tax resources from black yeoman to finance the education of white children. What constrains the white yeoman from extracting the entire black tax revenue is the potential for black yeoman to leave to a nondiscriminating district. This section is divided into two parts, the first details the problem facing white plantation owners and black sharecroppers, and the second section presents the problem facing white yeoman and black yeoman.

White Plantation Owners and Black Sharecroppers

This section analyzes the problem facing white plantation owners and black sharecroppers. White plantation owners choose the amount of their income to finance schooling of their children, as well as the consumption and educational expenditures on black sharecroppers and the children of black sharecroppers. Black sharecroppers decide whether to stay in the discriminating environment or migrate to a nondiscriminating district. Migration entails a cost and hence some black sharecroppers remain, despite the onerous effects of discrimination.

White plantation owners and black sharecroppers work together to produce output. Assume that the single output is produced by these two types of labor in combination with land, which is owned by the white plantation owner. We assume that the output of the representative white plantation owner is given by:

$$y_t = Z_t L_t^{\sigma} h_t^{\alpha} (n_{bt} h_{bt})^{1-\alpha-\sigma} \tag{1}$$

where Z_t is the Total Factor Productivity in production, L_t is the land holdings of the white plantation owner, h_t is the human capital of the white plantation owner, n_{bt} is the number of black sharecroppers working on the plantation and h_{bt} is the human capital of the typical black sharecropper. We assume that Z_t evolves as exogenous technological progress.[8] The production parameters satisfy, $1 > \alpha > 0$, $1 > \sigma > 0$, $\alpha + \sigma < 1$. This output specification allows that black sharecropper human capital is a productive input, and hence contributes to a plantation owner's willingness to provide educational opportunities for sharecroppers.

White plantation owners choose their own consumption, c_t, their expenditures on their children's education, X_t, and they monopsonistically provide their black sharecroppers compensation and educational provision, c_{bt}, X_{bt}. These are displayed in the following budget constraint facing the typical plantation owner:

$$c_t + X_t + n_{bt}(c_{bt} + X_{bt}) = y_t = Z_t L_t^{\sigma} h_t^{\alpha} (n_{bt} h_{bt})^{1-\alpha-\sigma} \tag{2}$$

We model each plantation as its own education district with the plantation owner as the only voter.

White plantation owners preferences are over their own consumption and the output available to their typical child. Their fertility is asexual and exogenous at g_w.

$$\ln c_t + \delta \ln y_{t+1} \tag{3}$$

From (2) it is clear that plantation owners wish to provide for the human capital of their children, and also for the human capital of the black sharecropper. We assume that there are two types of human capital for blacks, skilled, h_b^s, and unskilled, h_b.[9] Plantation owners are willing to provide unskilled human capital because it is directly productive in plantation production, but are less desirous of providing skilled human capital for the children of black sharecroppers, because it is only productive off of the

[8] We could have assumed that Z_t evolved exogenous to any individual, but endogenous in the sense of Romer (1986), Lucas (1988) or Tamura (1996,2002,2004). That is accumulation of human capital produces an external effect raising TFP. In the numerical solutions, we actually assume technological regress, $Z_{t+1} < Z_t$, which appears to accord with the declining soil productivity as well as the influence of the boll weevil.

[9] This is similar to the dichotomous human capital in Becker, Murphy and Tamura (1990).

plantation. Education produces unskilled human capital and as a joint product skilled human capital in fixed proportion. The human capital accumulation technology for black plantation children is similar to Tamura (2001) and given by:

$$\begin{aligned} h_{bt+1} &= Ah_{bt}\left(\frac{X_{bt}}{g_w h_t^T}\right)^{\varepsilon\nu} \\ h_{bt+1}^s &= \lambda h_{bt} \end{aligned} \tag{4}$$

where h_t^T is the human capital of the teacher. Observe that X_{bt} are resources spent hiring teachers, and the term in parentheses is class size for black sharecropper children. Since class size is the only input for accumulation, plantation owners will only hire amongst their plantation workers. In particular since class size is the only input that matters, hiring white teachers with greater human capital is a more costly method of producing unskilled human capital. We assume that white teachers are better at producing skilled human capital, but since this is not productive for the plantation owner, and increases the probability that tomorrow's sharecropper with leave the plantation, this is clearly not a desirable option.[10] In this case black teachers are paid like their fellow sharecroppers, c_{bt}. Thus since only blacks will be hired to teach black sharecropper children, class size will be given by:

$$\frac{s}{g_b} \tag{5}$$

where s is the proportion of sharecroppers hired as teachers.[11]

[10] Assume that hiring white adults with greater amounts of skilled human capital produces unskilled and skilled human capital in the following manner:

$$\begin{aligned} h_{bt+1} &= Ah_{bt}\left(\frac{X_{bt}}{g_w h_t^T}\right)^{\varepsilon\nu} \\ h_{bt+1}^s &= A\lambda h_{bt}\left(\frac{X_{bt}}{g_w h_t^T}\right)^{\varepsilon\nu}\left(\frac{h_t^T}{\lambda h_{bt}}\right)^{(1-\varepsilon)\nu} \end{aligned}$$

Observe that since $h_t^T > h_{bt}$, the white teacher produces less unskilled human capital than the black teacher, and more skilled human capital than the black teacher if $\varepsilon < \frac{1}{2}$, see Tamura (2001).

[11] The correct class size is $\frac{s}{g_b}$ and not $\frac{s}{(1-s)g_b}$ because black teachers have the same fertility as

The number of black sharecroppers per plantation is determined by the number of blacks who remain to work on the plantation. Some blacks may choose to migrate to an assumed nondisciminating district. Not only do they not suffer from discrimination in education provision, they also do not suffer from discrimination in production. Define the number of blacks who remain on the plantation as:

$$n_{bt} = N_{bt}\theta_t \tag{6}$$

where N_{bt} is the population of blacks per plantation at the start of period t, and θ_t is the proportion of blacks who choose to work on the plantation. We now turn to the determination of the fraction of blacks remaining on the plantation.

Plantation blacks face the choice of staying and working on the plantation and having their children receive education from the plantation district or pay a moving cost and migrate to a nondiscriminating district both in terms of production and in terms of educational opportunities for their children. If a black chooses to stay on the plantation his utility is given by:

$$U(stay) = \ln c_{bt} + \delta \ln h^s_{bt+1} \tag{7}$$

where the parent cares about both the consumption they receive as well as the skilled human capital of their children. Fertility is asexual and exogenous for blacks as well, g_b. If a black migrates from the plantation he pays a cost that is proportional to their human capital, chooses the rate of taxation on his income to pay for the public schooling of his children. We assume that each individual that chooses to move pays the same proportional cost to move. In other words the marginal individual determines the proportional cost of moving for all individuals who choose to move. Output produced by individuals not on the plantation is linear in their available human capital. Thus preferences are given by:

$$U(move) = \ln\left(h^s_{bt}\left[1-\varphi\right]\left[1-\tau_t\right]\right) + \delta \ln h^s_{bt+1} - f \tag{8}$$

black sharecroppers, and their children are educated on the plantation as well.

where φ is the marginal proportion of output that is foregone in order to move, τ is the tax rate to pay for schooling of their children and $f \geq 0$ is the psychic utility cost of migration. The decision whether to move or not therefore comes from whether the utility after the move is bigger or smaller than the utility of staying. In Appendix A. we assume that this proportional cost of moving is distributed uniformly across the black population:

$$\varphi \sim U[0,1] \tag{9}$$

Define the proportion of adult black sharecroppers that stay on the plantation in period t as θ_t. Since black teachers are hired from the sharecropper population, the white plantation optimization problem becomes:

$$\max_{\{X_t, c_{bt}, X_{bt}\}} \left\{ \begin{array}{c} \ln\left[Z_t L_t^{\sigma} h_t^{\alpha} (N_{bt}\theta_t h_{bt})^{1-\alpha-\sigma} - X_t - N_{bt}\theta_t c_{bt}\right] \\ +\delta \ln\left[Z_{t+1} L_{t+1}^{\sigma} h_{t+1}^{\alpha} (N_{bt}\theta_t \theta_{t+1} \frac{g_b}{g_w} h_{bt+1})^{1-\alpha-\sigma}\right] \end{array} \right\} \tag{10}$$

where land holdings evolve as:

$$L_{t+1} = \frac{L_t}{g_w} \tag{11}$$

that is to say, population growth of plantation owners reduces the amount of land holdings of their progeny.[12] Let the maximum human capital in the economy be given by $\overline{h}$. Appendix C. shows that the form of the stay probability is given by:

$$\theta_t = \min\left\{ \frac{c_{bt}^{\frac{1}{1+\delta\varepsilon\nu}} s_{bt}^{\frac{\delta\varepsilon\nu}{1+\delta\varepsilon\nu}} \left(\overline{h}_t\right)^{-\frac{\delta(1-2\varepsilon)\nu}{1+\delta\varepsilon\nu}} (1+\delta\varepsilon\nu) e^{\frac{f}{1+\delta\varepsilon\nu}}}{(\delta\varepsilon\nu)^{\frac{\delta\varepsilon\nu}{1+\delta\varepsilon\nu}} \left(h_{bt}^{s}\right)^{\frac{1-\delta(1-2\varepsilon)\nu}{1+\delta\varepsilon\nu}}}, 1 \right\} \tag{12}$$

The greater the consumption given to black sharecroppers the larger the proportion that stay, the greater the fraction of sharecroppers hired as teachers the greater the proportion of sharecroppers that stay. The greater the skilled human capital of black sharecroppers the more likely they are to leave. The better the available teachers in the economy, the more likely the sharecropper will leave. This causes the

[12] We abstract from primogeniture issues of inheritance.

plantation owner to reduce the amount of education to provide to his sharecroppers' children. The higher the psychic cost of moving the greater the proportion of stayers. Appendix C. deals with the case where $\theta_t < 1$. In the numerical solutions to come, $\theta_t = 1$ throughout the plantation era. Under this case we can solve for c_{bt} as a function of the share of black sharecroppers hired as teachers. This produces:

$$c_{bt} = s_{bt}^{-\delta\varepsilon\nu}(\delta\varepsilon\nu)^{\delta\varepsilon\nu}(1+\delta\varepsilon\nu)^{-(1+\delta\varepsilon\nu)}\left(h_{bt}^{s}\right)^{\frac{1-\delta(1-2\varepsilon)\nu}{1+\delta\varepsilon\nu}}\left(\overline{h}_t\right)^{\frac{\delta(1-2\varepsilon)\nu}{1+\delta\varepsilon\nu}} \tag{13}$$

In this case the first order conditions for the plantation owner are

$$\begin{aligned} \frac{1}{c_t} &= \frac{\alpha\delta\varepsilon\nu}{X_t} \\ \frac{(1-\alpha-\sigma)\,y_t}{c_t\,(1-s_{bt})} + \frac{N_{bt}\partial c_{bt}}{\partial s_{bt}} &= \frac{\delta\,(1-\alpha-\sigma)\,\varepsilon\nu}{s_{bt}} \end{aligned} \tag{14}$$

The first Euler equation states that a plantation owner equates the marginal cost of additional resources to educate their children with the marginal benefit of educational expenditures. The second Euler equation equates the marginal cost of smaller classes with the marginal benefit of smaller classes. It is easy to show that plantation owners spend a fixed fraction on their own consumption and education for their children out of available resources:

$$\begin{aligned} c_t &= \frac{1}{1+\alpha\delta\varepsilon\nu}\left\{y_t - N_{bt}c_{bt}\right\} \\ X_t &= \frac{\alpha\delta\varepsilon\nu}{1+\alpha\delta\varepsilon\nu}\left\{y_t - N_{bt}c_{bt}\right\} \end{aligned} \tag{15}$$

Appendix D. solves for the optimal share of black sharecroppers hired as teachers, s_{bt} as well as optimal consumption for black share croppers, c_{bt}.

White and black yeoman

In this subsection we analyze the problem facing the black and white yeoman districts. Black yeoman residing in a discriminatory district choose whether to remain in the district or to move into a nondiscriminating district. We show that unlike the plantation districts, black yeoman districts produce a constant rate of out migration. Consider the problem facing the typical white yeoman. Unlike his plantation owning counterpart, the white yeoman does not hire the black yeoman in production. Instead both whites and blacks produce separately from each other. Whites choose the tax rate on their income and the income of black yeoman in their district as well as the educational expenditures on both white children and black children. White and blacks have the same set of preferences, they care about both their adult consumption and the human capital of their progeny:

$$\ln c_t + \delta \ln h_{t+1} \tag{16}$$

Black yeomen face the choice of staying and working in a discriminatory environment or pay a moving cost and migrate to a nondiscriminating district both in terms of production and in terms of educational opportunities for their children. If a black chooses to stay on the plantation his utility is given by:

$$U(stay) = \ln c_{bt}^{y} + \delta \ln h_{bt+1}^{y} \tag{17}$$

where the parent cares about both the consumption they receive as well as the human capital of their children. Fertility is asexual and exogenous for blacks as well, $g_b \geq g_w$. If a black migrates from the discriminatory district he pays a cost that is proportional to their human capital, chooses the rate of taxation on his income to pay for the public schooling of his children. We assume that each individual that chooses to move pays the same proportional cost to move. In other words the marginal individual determines the proportional cost of moving for all individuals who choose to move. Output produced by individuals not on the plantation is linear in their available human capital. Thus preferences are given by:

$$U(move) = \ln \left(h_{bt} \left[1 - \varphi\right] \left[1 - \tau_t\right]\right) + \delta \ln h_{bt+1} - f \tag{18}$$

where φ is the marginal proportion of output that is foregone in order to move, τ is the tax rate to pay for schooling of their children and $f \geq 0$ is the psychic utility cost of migration. The decision whether to move or not therefore comes from whether the utility after the move is bigger or smaller than the utility of staying. In Appendix A. we assume that this proportional cost of moving is distributed uniformly across the black population:

$$\varphi \sim U[0,1]$$

For white yeomen, they set the tax rate on both their incomes and the black yeomen in their district, as well as the expenditures on the education of their children and the expenditures on the children of black yeoman. Income is linear in the human capital of the adult. Therefore the budget constraint for whites and blacks, and the budget constraint for the public school district with N_{bt}^{y} black yeoman per white yeoman, where the superscript denotes that it is a yeoman district, are:

$$\begin{aligned} c_t^y &= h_t^y\left(1-\tau_t^y\right) \\ c_{bt}^y &= h_{bt}^y\left(1-\tau_{bt}^y\right) \\ X_t^y + N_{bt}^y\theta_t^y X_{bt}^y &= h_t^y\tau_t^y + h_{bt}^y\tau_{bt}^y N_{bt}^y\theta_t^y \end{aligned} \tag{19}$$

Since whites do not care about the human capital of the children of black yeoman, they only care how much they can extract from black yeoman in order to pay for the education of their children. We assume that the extraction cannot be used to directly support consumption of white yeoman. Holding the amount spent on the education of the children of black yeoman constant, X_{bt} constant, hiring the best teachers produces the higher proportion of stayers in the black yeoman population, greater θ_t^y. This proves the following proposition:

Proposition 1 *It is optimal to hire the best possible teachers for both the white children and the black children.*

The yeoman district differs from the white plantation district in that black students receive a much higher skilled teacher.

Thus human capital in the next period for white and black yeoman are:

$$
\begin{aligned}
h_{t+1}^{y} &= Ah_{t}^{y}\left(\frac{X_{t}^{y}}{g_{w}h_{t}^{T}}\right)^{\varepsilon\nu}\left(\frac{h_{t}^{T}}{h_{t}^{y}}\right)^{(1-\varepsilon)\nu} \\
h_{bt+1}^{y} &= Ah_{bt}^{y}\left(\frac{X_{bt}^{y}}{g_{b}h_{t}^{T}}\right)^{\varepsilon\nu}\left(\frac{h_{t}^{T}}{h_{bt}^{y}}\right)^{(1-\varepsilon)\nu} \qquad (20)
\end{aligned}
$$

With these results and ignoring terms not involving choice variables, the problem facing the typical white yeoman is:

$$
\max_{\left\{\tau_{t}^{y},\tau_{bt}^{y},X_{bt}^{y}\right\}}\left\{\ln\left(1-\tau_{t}^{y}\right)+\delta\varepsilon\nu\ln\left[h_{t}^{y}\tau_{t}^{y}+h_{bt}^{y}\tau_{bt}^{y}N_{bt}^{y}\theta_{t}^{y}-N_{bt}^{y}\theta_{t}^{y}X_{bt}^{y}\right]\right\} \qquad (21)
$$

where the proportion of black yeoman that stay in the discriminating district is given by and ignoring the y superscript:

$$
\theta_{t}^{y}=c_{bt}^{\frac{1}{1+\delta\varepsilon\nu}}X_{bt}^{\frac{\delta\varepsilon\nu}{1+\delta\varepsilon\nu}}h_{bt}^{-\frac{\delta\varepsilon\nu}{1+\delta\varepsilon\nu}}(1+\delta\varepsilon\nu)(\delta\varepsilon\nu)^{\frac{-\delta\varepsilon\nu}{1+\delta\varepsilon\nu}}e^{\frac{f}{1+\delta\varepsilon\nu}} \qquad (22)
$$

Notice that the functional form of the proportion of stayers in the black yeoman population is identical to that found in the black plantation district. Inserting this into the white yeoman's preferences and differentiating with respect to the three control variables produces the following Euler equations:

$$
\begin{aligned}
\frac{1}{1-\tau_{t}^{y}} &= \frac{\delta\varepsilon\nu h_{t}^{y}}{X_{t}^{y}} \\
\frac{\delta\varepsilon\nu N_{bt}^{y}\theta_{t}h_{bt}^{y}}{X_{t}^{y}}+\frac{\delta\varepsilon\nu}{X_{t}^{y}}\left\{N_{bt}^{y}h_{bt}^{y}\tau_{bt}^{y}-X_{bt}^{y}N_{bt}^{y}\right\}\frac{\partial\theta_{t}}{\partial\tau_{bt}^{y}} &= 0 \\
-\frac{\delta\varepsilon\nu N_{bt}^{y}\theta_{t}}{X_{t}^{y}}+\frac{\delta\varepsilon\nu}{X_{t}^{y}}\left\{N_{bt}^{y}h_{bt}^{y}\tau_{bt}^{y}-X_{bt}^{y}N_{bt}^{y}\right\}\frac{\partial\theta_{t}}{\partial X_{bt}^{y}} &= 0 \qquad (23)
\end{aligned}
$$

Unlike in the case of the plantation district, these Euler equations regarding the white tax rate, the black tax rate and the black expenditures on education can be easily solved, the details are contained in Appendix C. We present the results here:

$$\begin{aligned}
\tau_{bt}^{y} &= \frac{1+2\delta\varepsilon\nu}{2\left(1+\delta\varepsilon\nu\right)} \\
X_{bt}^{y} &= \frac{\delta\varepsilon\nu}{2\left(1+\delta\varepsilon\nu\right)} h_{bt}^{y} \\
\tau_{t}^{y} &= \max\left\{0, \frac{\delta\varepsilon\nu}{1+\delta\varepsilon\nu} - \frac{N_{bt}^{y}\theta_{t}h_{bt}^{y}}{2\left(1+\delta\varepsilon\nu\right)h_{t}^{y}}\right\} \\
\theta_{t}^{y} &= \frac{e^{\frac{f}{1+\delta\varepsilon\nu}}}{2}
\end{aligned} \tag{24}$$

One surprising result is that the stay probability is equal to 50 percent if there are no psychic costs of moving. Tax rates for black yeoman are much larger than they would be in a nondiscriminating district. The relative tax rate is given by:

$$\frac{\tau_{bt}^{y}}{\tau_{t}} = 1 + \frac{1}{2\delta\varepsilon\nu} \tag{25}$$

Under a calibrated version of the model the tax rate for education would equal the share of resources spent on education. In the US public and private expenditures on K-12 and higher education relative to GDP in 2001 amounts to (392+30+277)/10208 = .068. In a nondiscriminating district the optimal tax rate is given by $\frac{\delta\varepsilon\nu}{1+\delta\varepsilon\nu}$. This produces an estimate of $\delta\varepsilon\nu = .073$. Replacing this into (24) implies that the discriminatory tax rate for black yeomen is 7.85 times greater than the nondiscriminating tax rate. If tax rates were identical, but the form of the discrimination took the form of differential property value assessment, this requires that black property was assessed at 7.85 times the value of a nondiscriminating district. While blacks are being taxed at a much higher rate than their tax rate in a nondiscriminating district, they receive less than they pay in. Taking the ratio of the expenditures to tax revenues produces:

$$\frac{X_{bt}^{y}}{\tau_{bt}^{y}h_{bt}^{y}} = \frac{\delta\varepsilon\nu}{1+2\delta\varepsilon\nu} \tag{26}$$

Again, using the calibrated value for $\delta\varepsilon\nu$ implies that for every dollar paid in taxes, blacks receive only 6.4 cents! Thus white yeoman are able to divert almost 95 cents of every dollar received from black tax payers for white children's education![13]

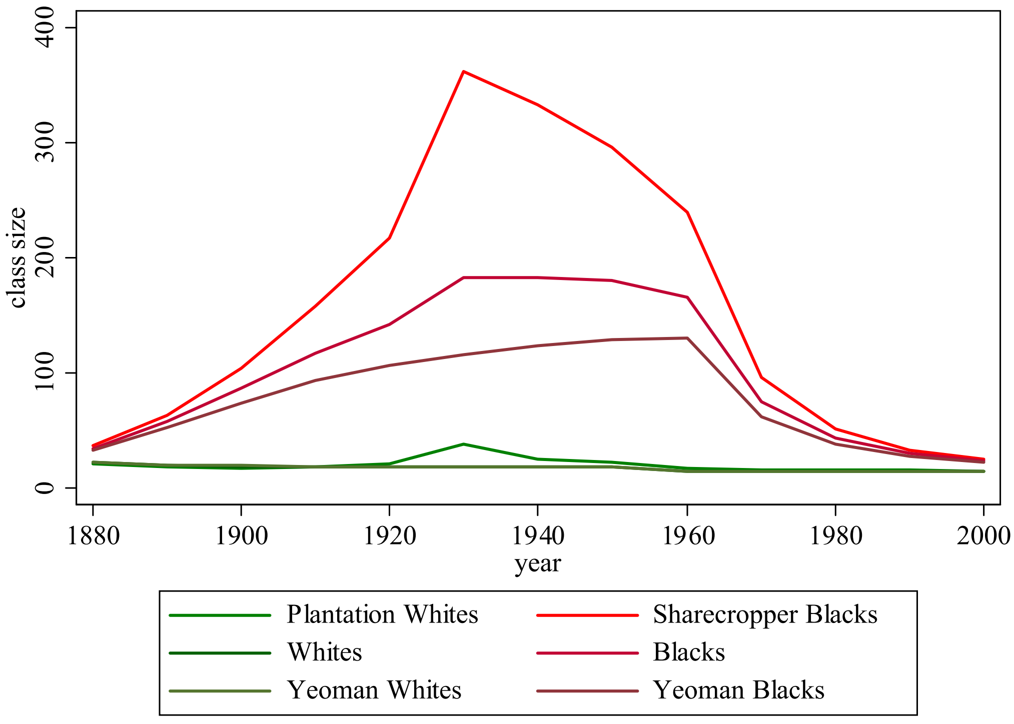
NUMERICAL SOLUTIONS AND COMPARISON OF PLANTATIONS AND TOWNS

In this section we numerically solve for the time paths of the human capital of white plantation owners, white yeomen, black sharecroppers and black yeomen. We construct two types of counties. In the first county all blacks are initially sharecroppers on identical plantations. The second county contains black yeomen. Furthermore we compare the relative class sizes between these two types of counties. In order to present the material in a relatively clean manner, we calculate class size, for each district, plantation whites, blacks in plantation counties (some are sharecroppers and some are in nondiscriminating districts), white yeoman and blacks in yeoman counties (some are in discriminating districts and others are in nondiscriminating districts). In our solution we must choose parameters for δ, ε, ν, σ, α, λ, A, g_b, g_w, and f. For land's share of output we assume that $\sigma = .15$. Furthermore we assume that α, the share of output that the plantation owner would receive in a competitive world is $\frac{3}{4}$. These two parameters imply that plantation owners would receive 90 percent of output from the plantation in a competitive economy. This places an extreme value on the income inequality of white plantation owners relative to their black sharecroppers, even in a competitive economy. While we do not appeal to micro evidence for many of these parameters, we pick parameters in order to replicate some observable quantities. As mentioned earlier, average class size converges to the value $\frac{1+\delta\varepsilon\nu}{\delta\varepsilon\nu g}$, where g is the growth rate of population. In steady state we assume $g_b = g_w = 1.01$. From

[13] Data for 1920 from the plantation county Calhoun county shows an extraction rate of more than 33 percent. Hampton county, another plantation county may have had an extraction rate of more than 50 percent by 1920. For more on this see Canaday (2003). These suggest that it is possible that yeoman county extraction rates were extremely large.

Baier, Mulholland, Tamura and Turner (2004), average class size for South Carolina in 1999 was 14.7. Average spending rate on education in the model converges to $\frac{\delta\varepsilon\nu}{1+\delta\varepsilon\nu}$. For the US total public and private spending per K-12 and higher education relative to GDP in 2001 was .068. The data on black and white population in South Carolina from 1880-2000 provides information on relative population growth rates of blacks and whites. We assume that white population grows at a rate of 1.18 percent per year and black population grows at a rate of .25 percent per year from 1880 until 1950. After 1950 we assume that both populations grow at the rate of 1 percent per year. In our model we assume that a period is 10 years. Table 3 presents the parameters as well as the calibrated steady state class size, actual class size and rate of spending on education. The demographic characteristics are given in the bottom panel of Table 3. We assume that initially there are two types of blacks in each county, those that suffer from discrimination and those that do not. Table 4 provides the share of black population in South Carolina from 1880 to 2000. The fit, although not exact, we believe is reasonable. There is an incentive compatibility constraint on white plantation owners. Since plantation whites can always switch to produce output like yeoman whites (sharecropper blacks can always produce as yeoman blacks), we assume that when yeoman white production with discrimination produces higher utility than the plantation system, white plantation owners switch. This switch occurs in the numerical solutions below in 1920. From 1960 onward we assume that all discriminatory power of whites disappears. We model this as all districts become competitive suppliers of education to both whites and blacks and an equalization of the rate of return to educational expenditures, $A_b = A_w$.

The first figure contains our solution for class size.

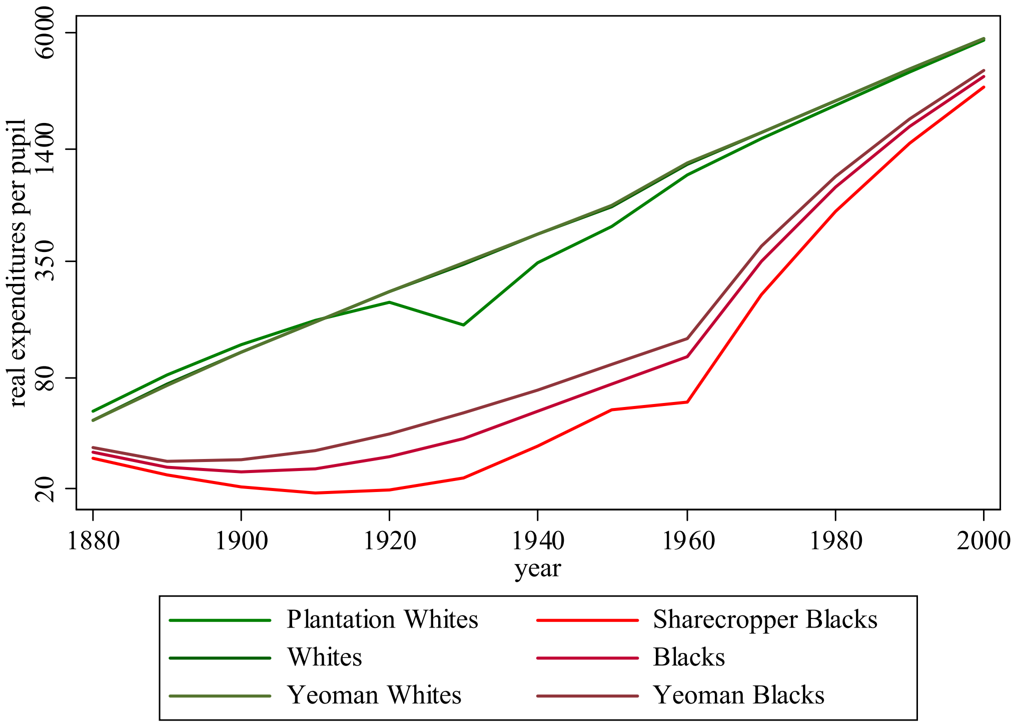


Class size

The model has sharp reductions in black class size only from 1960 onward. However notice that black class size is essentially constant from 1880 to 1960, with the exception of sharecropper blacks. The sharp reduction in class size both for whites as well as blacks after 1960 is consistent with the data, however the lack of decline for blacks prior over the 1920-1960 period is at odds with the data. Sharecropper blacks have larger class size than yeoman blacks, which is at variance with the data presented in Figure 1 for the period prior to 1920.[14]

The next figure presents our results for average expenditures per pupil.

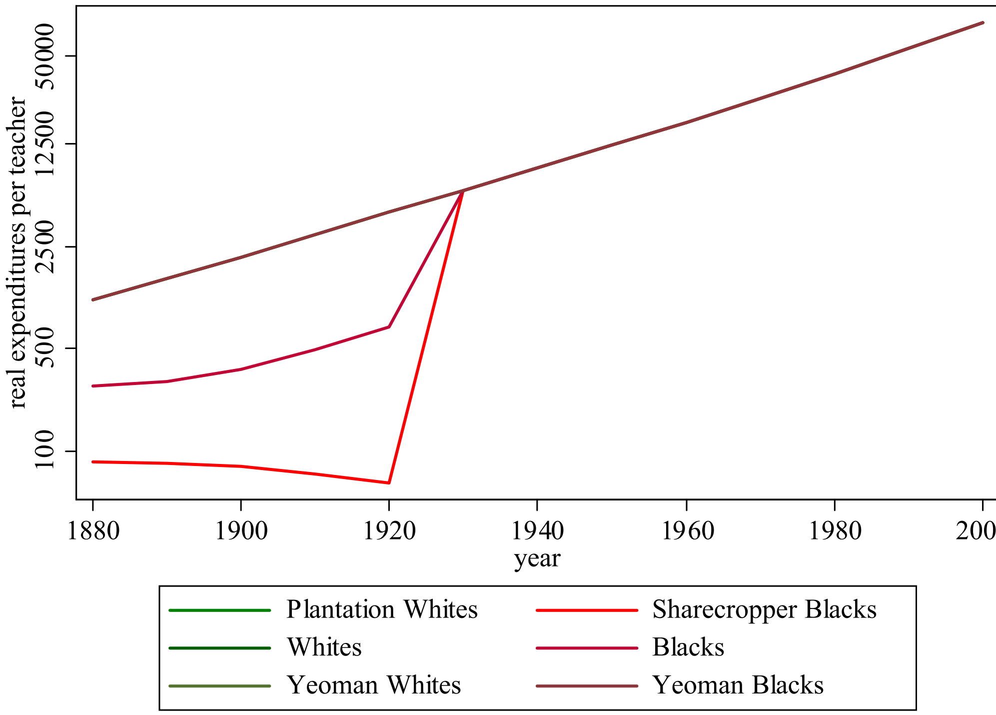
[14]In Figure 1, we graphed average class size for whites and blacks for the entire state. For the Plantation counties and Yeoman counties we only used information on a subset of counties in the state in order to get a general feel for the data.



Real expenditures per pupil

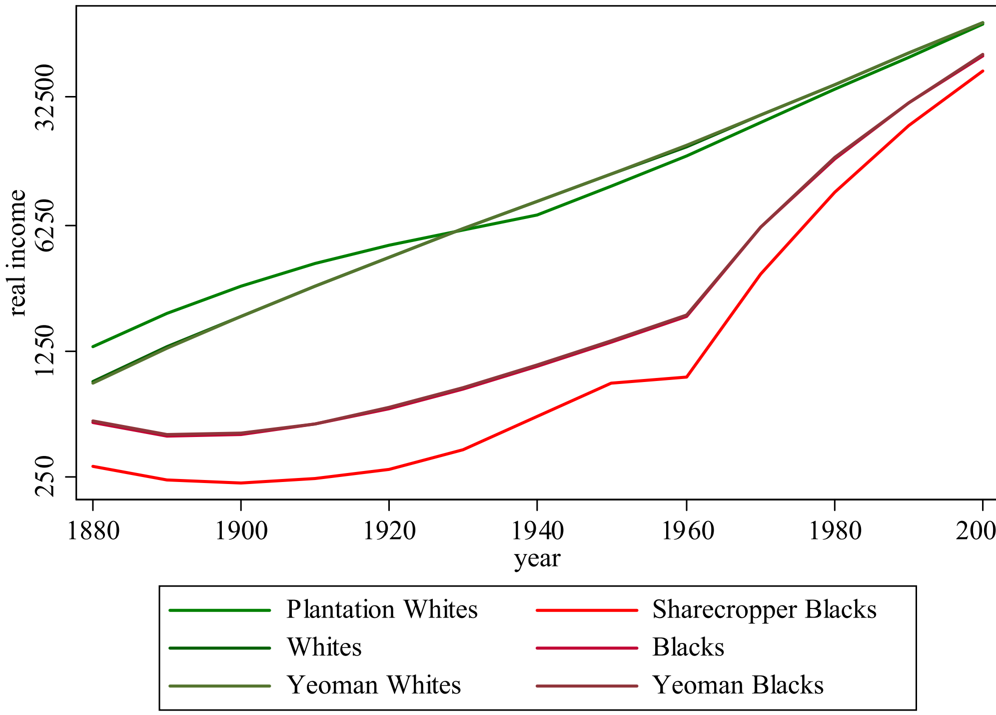
The model was calibrated so that average spending per white pupil in the solution was equal to real spending on white pupils in South Carolina in 1900 and 1960. Black expenditures per pupil in 1900 and 1960 are 27 dollars and 734 dollars. White expenditures per pupil in 1900 and 1960 are 88 dollars and 1128 dollars. The model solution here produces white expenditures of 88 dollars and 1129 dollars respectively. However for black expenditures the model produces values of 27 dollars and 301 dollars. Clearly the model underpredicts the growth of black expenditures per pupil.

The next graph presents real expenditures per teacher.



Real expenditures per teacher

We observe white and black income from 1940-2000 via the censuses, we present the average real income for whites and blacks by county type and for the state as a whole from 1880-2000. The relative gap between whites and blacks rises steadily from the end of Reconstruction in 1880 to 1960. With the federal intervention the model indicates rapid convergence in human capital.



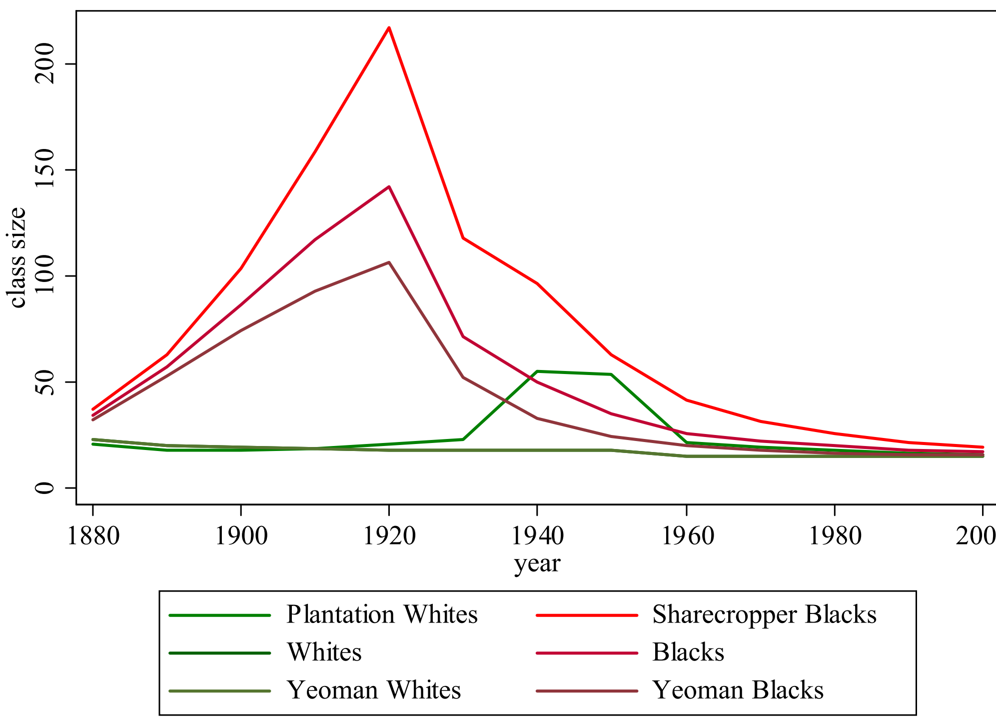
Real income

WORLD WAR I

This section introduces the effect of World War I on black educational outcomes. A reexamination of figures 1-3 indicate that the period from 1880-1920 was one of relative stagnation in black class size, and from 1900-1920 stagnation in black real spending per pupil and black real spending per teacher. Closer examination indicates a break in all three of these series around the conclusion of World War I. One possible connection is that World War I had a dramatic effect on immigration from Europe to the United States over this period. Table 5 contains the level of international migration to the United States from 1900-1964 in five year intervals. It is clear that the onset of World War I had a dramatic reduction in the rate of immigration from Europe to the United States. From 1900-1914 international immigration averaged 890 thousand per year and European immigration averaged 811 thousand per year. In the following 15 years, 1915-1929, international immigration averaged 365 thousand per year and European immigration averaged only 207 thousand per year. The Great Depression and World War II also reduced immigration, as only 60 thousand international immigrants arrived per year and only 36 thousand European immigrants arrived per year during the 1930-1944 period. Northern employers recruited southern workers to move north to fill the demands of firms; this effect was documented by Whatley (1990). This period also coincides with the advertisement for black workers in industrial cities of the midwest, Grossman (1989). In this section we provide a measure of this importance by modifying the model of the paper to include improved conditions for blacks. The solution in the previous section clearly does not fit this stagnant period prior to the rapid increase in black educational inputs.

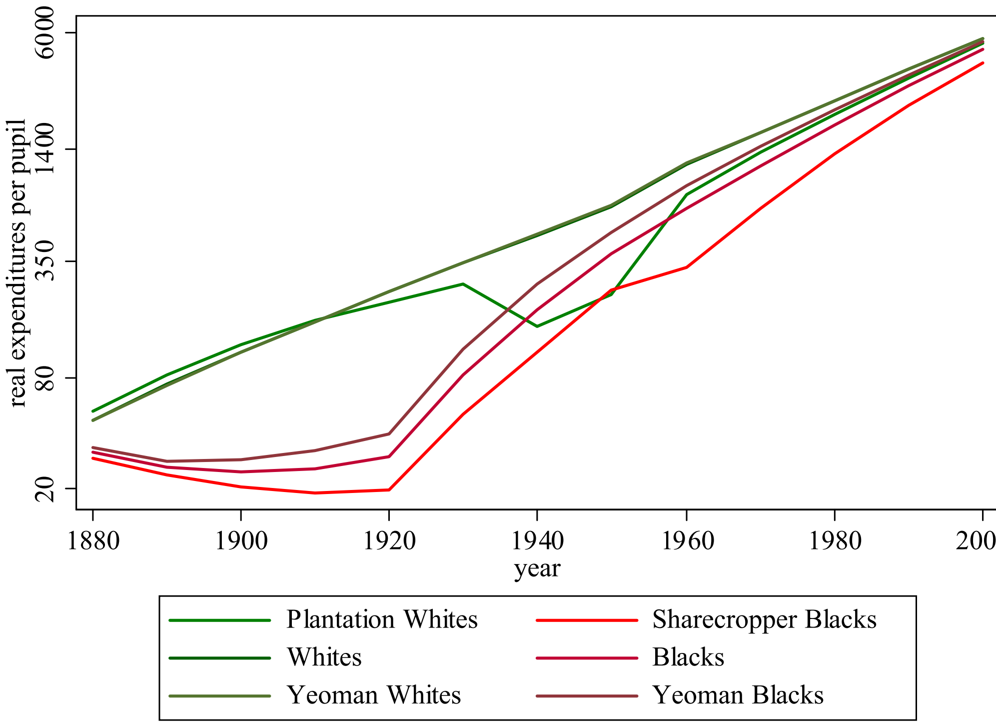
We incorporate this by assuming that the efficiency of black education, A_b, changes after World War I. In particular we assume that moving from $\frac{A_b}{A_w} = .56$ prior to 1920 to $\frac{A_b}{A_w} = .85$ from 1920 onward until 1960. In 1960 we assume as before that $A_b = A_w$. Additionally we assume that improved information about alternative locations reduces the psychic cost of migration for both sharecropper blacks and yeoman blacks. Thus we assume that $f = 0$ from 1920 onward. The reason we do these modifications is that imposing parameter restrictions on $\delta\varepsilon$ in order to fit current class size and current expenditures on education implies a low elasticity of migration with respect to educational expenditures.

The model is able to produce the relative constancy of black class size at the state level from 1880 to 1920. It matches the decline in class size after 1920 quite well.



Class size

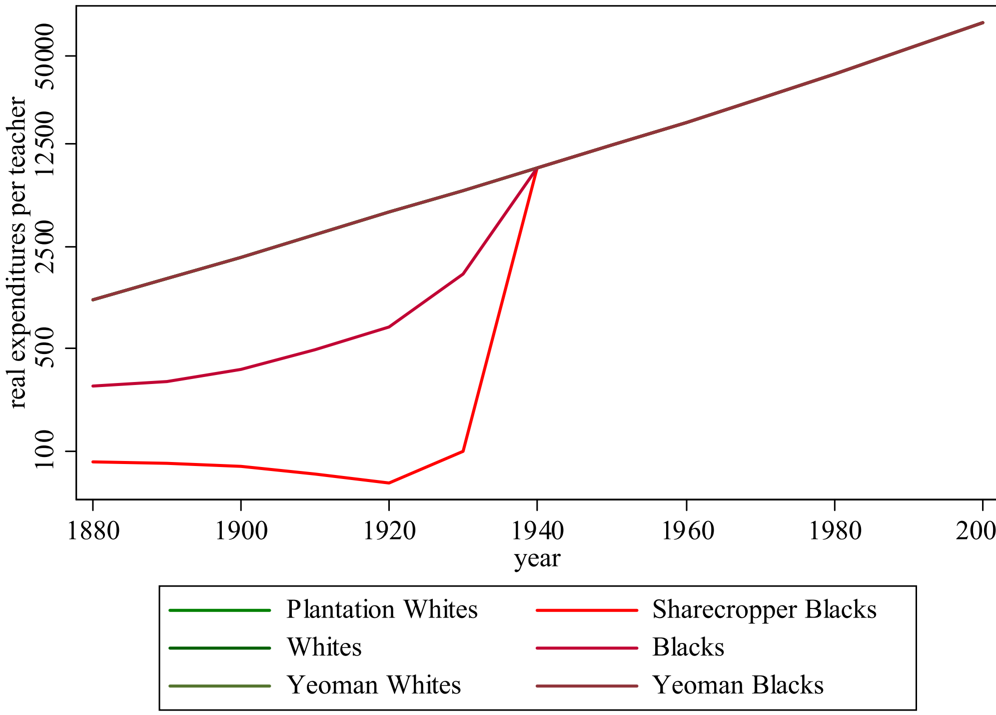
Spending per pupil for blacks and whites are given in the next figure. It is quite clear that black spending per pupil spending jumps discretely relative to whites after 1920. The model is capable of reproducing the values of black and white spending per pupils in 1900 and 1960. White spending per pupil rises from 88 dollars to 1130 dollars between 1900 and 1960, fitting the data. For blacks, spending per pupil rises from 29 dollars in 1900 to 798 dollars in 1960. One difference is the rise in spending per black pupil between 1900 and 1920, that is counterfactual. Table 6 contains the results for this model as well as the data for South Carolina blacks and whites from 1900-1960. While not fitting the data for blacks as well as for whites, the model with World War I effects clearly dominates the model without World War I effects for black expenditures. The 1950 dip in real expenditures per pupil for plantation whites compared with yeoman blacks and blacks as a whole occurs because the plantation owners switch to yeoman production in this period.



Real expenditures per pupil

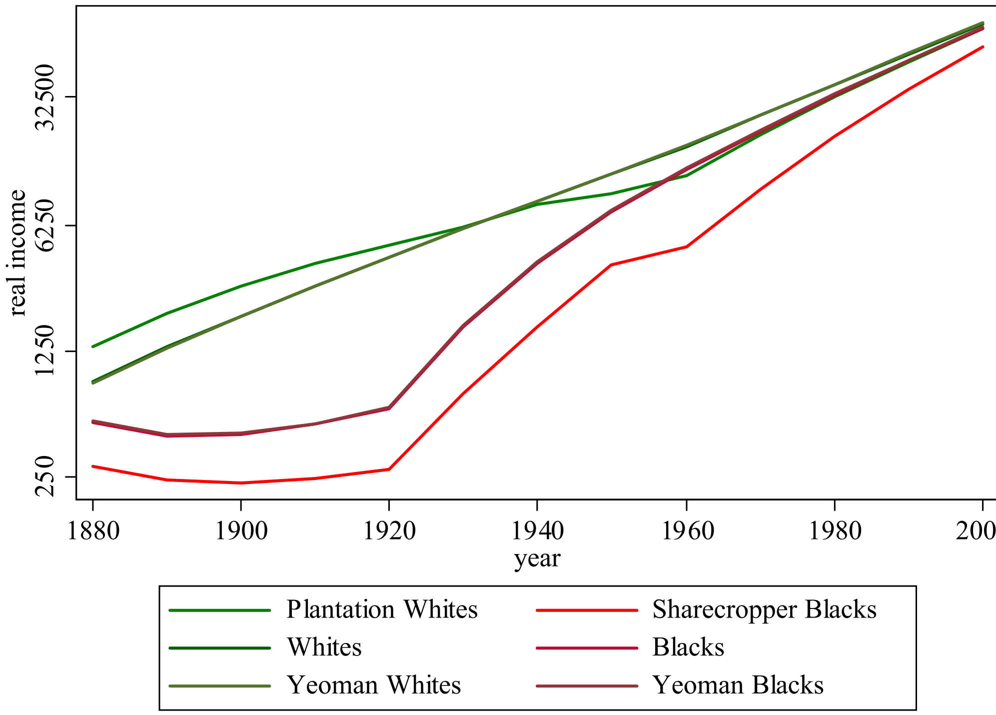
Spending per teacher is given by the next graph. Table 7 contains the comparison of the model with World War I vs. the model without World War I with the data from South Carolina. The model does a fairly good job at reproducing the time series of expenditures per teacher, although the levels are not exactly fit.[15] It is clearly obvious that from 1940-1960 the behavior of real expenditures per teacher in the model with World War I is much better than the model without World War I.

[15]One obvious difference between the model and the data is the lack of capital expenditures in the model. If over the period 1900-1960 labor received about 50 percent of the total expenditures per student, then the levels are fit quite well.



Real expenditures per teacher

The South Carolina real income series for whites and blacks are given in the next figure.



Real income

Table 8 provides a summary of the two different model solutions as well as evidence on relative black white earnings. The latter data come from Smith and Welch (1989), Smith (1993) and Couch and Daly (2000). The model without a World War I effect produces too little income convergence, and it only arrives after 1960. In contrast the model with a World War I effect provides too much income convergence, although it clearly indicates convergence in income from 1930 onward. Part of the problem lies with the comparison group. In the data, black income is for all blacks, not just those in the south, and white income is for all white income, not just for those in the south. Furthermore the absence of strong convergence in relative black income since 1980 is consistent with the evidence from Hoxby (1996). That is to say the declining returns to measured school inputs of class size and relative teacher quality that is found in Hanushek (1986), rationalized by Hoxby (1996), and contrasted with Card and Krueger (1992) and Tamura (2001), indicate a structural change in education production.[16]

CONCLUSION

This paper produces a model of human capital accumulation of whites and blacks in the presence of discrimination. There are two types of school districts, plantation districts and yeoman districts. In the plantation district, white plantation owners are monopsony employers of black sharecroppers and discriminating providers of black education. In the yeoman district, white yeoman are discriminating providers of black education. Migration is costly however it is not prohibitive. This migration provides a reservation utility for black sharecroppers and black yeoman. The model indicates that black mobility and teacher quality, during the period of Jim Crow discrimination in South Carolina, was sufficient to offset the tremendous levels of discrimination facing blacks. That is despite overwhelming disadvantages, blacks were able to achieve human capital growth during Jim Crow discrimination between 1880-1920. We also find that World War I appears to have been responsible for a break in the black white relationship. Improved information about outside alternatives for blacks provide the impetus for even more rapid human capital accumulation for blacks prior to the federal civil rights interventions: *Brown vs. Board of Education* (1954) and the *Voting Rights Act* (1964). The end of plantation production if coupled with the elimination in discrimination would increase the rate of convergence dramatically. We plan on comparing these post plantation production results with one that allows plantation owners to become discriminating white yeoman district. If 1960 or 1950 are viewed as the dates of Federal intervention, perhaps the augmented model can replicated ovservations on black white income differences achieved without Federal intervention.

[16]This would be consistent with a rise in the relative importance of class size, ε, as well as a reduction in overall inputs productivity, ν.

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Table 1: South Carolina Population

year	*population* (*thousands*)	black share	rate of growth: SC	rate of growth: US
1790	249	.438	-	-
1800	345	.432	.326	.301
1810	415	.484	.185	.310
1820	502	.528	.190	.286
1830	581	.556	.146	.289
1840	594	.564	.022	.283
1850	669	.589	.119	.307
1860	703	.586	.050	.304
1870	706	.589	.004	.236
1880	995	.607	.343	.231
1890	1151	.599	.146	.227
1900	1340	.584	.152	.191
1910	1515	.552	.123	.191
1920	1684	.514	.106	.139
1930	1738	.457	.032	.150
1940	1898	.429	.088	.070
1950	2115	.389	.108	.135
1960	2380	.348	.118	.170
1970	2583	.305	.082	.126
1980	3121	.304	.189	.108
1990	3447	.302	.107	.093
2000	4012	.295	.152	.124

Table 2: Southern State Black Share

year	*bshare* *AL*	*bshare* *GA*	*bshare* *LA*	*bshare* *MS*	*bshare* *NC*	*bshare* *SC*	*bshare* *VA*
1790		.361			.269	.438	.442
1800		.368		.500	.293	.432	.454
1810		.425	.545	.548	.322	.484	.485
1820	.328	.443	.523	.440	.344	.528	.496
1830	.384	.426	.583	.482	.359	.556	.498
1840	.433	.411	.551	.524	.357	.564	.490
1850	.447	.425	.506	.512	.364	.589	.471
1860	.454	.441	.494	.552	.365	.586	.450
1870	.477	.460	.501	.536	.366	.589	.419
1880	.475	.470	.515	.574	.379	.607	.418
1890	.448	.468	.500	.576	.347	.599	.383
1900	.452	.467	.471	.585	.329	.584	.357
1910	.425	.451	.431	.561	.316	.552	.325
1920	.384	.416	.389	.522	.298	.514	.299
1930	.357	.368	.369	.502	.290	.457	.268
1940	.347	.347	.359	.492	.275	.429	.247
1950	.320	.309	.329	.453	.258	.389	.221
1960	.300	.285	.319	.421	.245	.348	.206
1970	.262	.259	.299	.368	.222	.305	.185
1980	.256	.268	.294	.352	.224	.304	.189
1990	.253	.270	.308	.356	.220	.302	.188
2000	.260	.287	.325	.363	.216	.295	.196

Table 2 (cont'd): Black Share of Population and Share of Blacks in South

year	$\frac{bshare}{US}$	Share of black population in the South
1790	.193	.733
1800	.187	.733
1810	.189	.739
1820	.184	.757
1830	.181	.775
1840	.168	.791
1850	.157	.806
1860	.141	.823
1870	.127	.807
1880	.131	.826
1890	.119	.817
1900	.116	.814
1910	.107	.807
1920	.099	.770
1930	.097	.707
1940	.098	.690
1950	.100	.602
1960	.106	.523
1970	.111	.451
1980	.118	.454
1990	.123	.453
2000	.123	.473

Table 3: Parameter Values and Steady State Solutions vs. Data

parameter	base case	World War I	
δ	.31875	.31875	
ε	.286	.286	
ν	.80	.80	
σ	.15	.15	
α	.75	.75	
λ	.21	.21	
A_w	2.75	2.75	
A_b			
t $<$ 1960	$A_w * .45$	$A_w * .45$	t $<$ 1920
t $\geq$ 1960	A_w	A_w	1920 $\leq$ t
f			
t $<$ 1960	.1	.1	t$<$1920
t $\geq$ 1960	0	0	1920 $\leq$ t
	white pop	black pop discriminated	black pop not discriminated
Plantation county	1	16	9
Yeoman county	33	5.808	19.8
	white h.c.	black h.c. discriminated	black h.c. not discriminated
Plantation county	40	2.73	22
Yeoman county	30.08	14	22
	solution steady states	data	
class size	14.7	14.7	
education share	.0685	.0685	

Table 4: Black Share Model Solution Compared to Data

year	Model Solution	Data
1880	.598	.607
1890	.561	.599
1900	.523	.584
1910	.484	.552
1920	.446	.514
1930	.406	.457
1940	.365	.429
1950	.331	.389
1960	.304	.348
1970	.304	.305
1980	.304	.304
1990	.304	.302
2000	.304	.295

Table 5: Immigration to the United States: 1900-1950 (millions)[17]

year	total immigration	total European immigration
1900-1904	3.255	3.095
1905-1909	4.947	4.539
1910-1914	5.175	4.524
1915-1919	1.173	0.532
1920-1924	2.775	1.787
1925-1929	1.521	0.789
1930-1934	0.427	0.260
1935-1939	0.272	0.186
1940-1944	0.204	0.098
1945-1949	0.653	0.376
1950-1954	1.099	0.717
1955-1959	1.400	0.690
1960-1964	1.419	0.550

[17] Series C89-119 "Immigrants, by Country: 1820-1970," *Historical Statistics of the United States: Colonial Times to 1970*.

Table 6: Black-White Real Spending Per Pupil

year	white no WWI	black no WWI	white WWI	black WWI	white data	black data
1900	111	25	111	25	111	27
1910	163	26	163	26	179	34
1920	236	30	236	30	212	28
1930	336	38	339	84	568	85
1940	482	53	477	189	543	132
1950	690	75	681	380	721	348
1959	1168	105	1161	670	1128	734

Table 7: Black-White Real Spending Per Teacher

year	white no WWI	black no WWI	white WWI	black WWI	white data	black data
1900	2127	365	2127	365	4478	1937
1910	3036	488	3036	488	6425	2406
1920	4280	701	4280	701	7513	1999
1930	5995	5995	5995	1623	15846	4078
1940	8552	8552	8552	8552	15362	5059
1950	12199	12199	12199	12199	19841	11256
1959	17401	17401	17402	17402	31750	24445

Table 8: Black-White Human Capital Ratios & Black-White Earnings Ratios[18]

[18] Smith and Welch (1989) comes from Table 8 of their paper. To calculate the average value I took the observations centered around the year in the Table and averaged throughout their data. For example the value of .56 for 1940 comes from examining the relative wages of blacks whose median year of entry into the labor force was 1938 and 1943 and relative wage data covers the years 1940, 1950, 1960 and 1970. For Smith (1993) the 1990 value comes from an arithmetic average of his 1989 entry in the first two columns. Couch and Daly (2000) provides information for 1998 on black male workers with less than 6 years of work experience.

year	model no WWI	model WWI	Smith-Welch	Smith (1993)	Couch & Daly
1880	.59	.59			
1890	.32	.32			
1900	.22	.22	.40		
1910	.17	.17	.41		
1920	.14	.14	.48		
1930	.13	.29	.52		
1940	.12	.45	.59		
1950	.12	.62	.63		
1960	.12	.75	.67		
1970	.24	.82	.75		
1980	.39	.88	.84		
1990	.53	.92		.85	
2000	.66	.94			.88

A. BLACK SHARECROPPER MIGRATION PROBABILITY

If a black sharecropper chooses to stay on the plantation and white plantation owners hire the black sharecroppers to teach black sharecropper children, then the sharecropper's utility is:

$$\ln c_{bt} + \delta \ln \left[A\lambda h_{bt} \left(\frac{s_{bt}}{g_b} \right)^{\varepsilon\nu} \right]$$

If the blacksharecropper moves to a nondiscriminating district his utility is given by:

$$\ln \left[\lambda h_{bt} \left(1 - \varphi \right) \left(1 - \tau_t \right) \right] + \delta \ln \left[A\lambda h_{bt} \left(\frac{\tau_t \lambda h_{bt} \left[1 - \varphi \right]}{g_b h_t^T} \right)^{\varepsilon\nu} \left(\frac{h_t^T}{\lambda h_{bt}} \right)^{(1-\varepsilon)\nu} \right] - f$$

In the model we assume that the proportional cost of moving $\varphi \sim U[0,1]$. The draws are i.i.d. across individuals, however the proportional cost paid by all movers is given by the marginal mover. Solving for the optimal tax rate in the nondiscriminating district produces the following result:

$$\tau_t = \frac{\delta\varepsilon\nu}{1 + \delta\varepsilon\nu}$$

Observe that the optimal tax rate is independent of the level of human capital of the individual. Thus the blacks choice of tax rate is unanimous.

Equating utilities and simplifying produces:

$$\ln c_{bt} + \delta\varepsilon\nu \ln s_{bt} = \left(1 - \delta \left(1 - 2\varepsilon \right) \nu \right) \ln h_{bt}^s + \delta \left(1 - 2\varepsilon \right) \nu \ln \overline{h}_t + \left(1 + \delta\varepsilon\nu \right) \ln \left[1 - \varphi \right] + \ln \left[1 - \tau_{bt} \right] + \delta\varepsilon\nu \ln \tau_{bt} -$$

where $h_t^T = \overline{h}_t = \max\{h_t\}$, since $\varepsilon < \frac{1}{2}$. Substituting for τ_{bt} and solving for $1 - \varphi$, the probability of staying on the plantation produces:

$$\theta_t = \min\{1 - \varphi_t, 1\} = \min\{ c_{bt}^{\frac{1}{1+\delta\varepsilon\nu}} s_{bt}^{\frac{\delta\varepsilon\nu}{1+\delta\varepsilon\nu}} \left(h_{bt}^s \right)^{-\frac{(1-\delta(1-2\varepsilon)\nu)}{1+\delta\varepsilon\nu}} \overline{h}_t^{\frac{-\delta(1-2\varepsilon)\nu}{1+\delta\varepsilon\nu}} (1+\delta\varepsilon\nu)(\delta\varepsilon\nu)^{\frac{-\delta\varepsilon\nu}{1+\delta\varepsilon\nu}} e^{\frac{f}{1+\delta\varepsilon\nu}}, 1\}$$

B. IMPORTANCE OF TWO TYPES OF HUMAN CAPITAL

In this formulation we show that the assumption of two types of human capital is crucial for the result of hiring black teachers. Suppose that there was only one type of human capital. Furthermore assume that human capital is accumulated as:

$$h_{t+1} = Ah_t \left(\frac{X_t}{g_b h_t^T} \right)^{\varepsilon\nu} \left(\frac{h_t^T}{h_t} \right)^{(1-\varepsilon)\nu}$$

This technology allows for either of two different regimes, either the district hires the highest human captial adults to be teachers, or the lowest human capital adults to be teachers. In this paper we assume that the teacher population comes from either within a plantation district or from the yeoman district. In other words we ignore the possibility of importing teachers from outside of the state. Tamura (2001) shows that which hiring regime is chosen simply depends on the magnitude of ε. If $\varepsilon < \frac{1}{2}$, then human capital is maximized for a given expenditure, X, by hiring the highest human capital adults to be teachers. If on the other hand $\varepsilon > \frac{1}{2}$, then the mazimizing choice is to hire only the lowest human capital adults to be teachers. Thus the regime choice depends on the relative importance of class size versus relative teacher quality.

Assume $\varepsilon < \frac{1}{2}$. Perhaps surprisingly plantation owners decide to use the same quality of teachers for black sharecropper children as they do for their own children. Furthermore the teacher quality chosen by blacks in the nondiscriminating district is the same as if they stayed, of course class sizes differ. We state this as the following proposition:

Proposition 2 *White plantation owners choose to employ the same quality teachers for the children of their black sharecroppers as for their own children. Furthermore these teachers are the same quality as teachers black parents would choose if they lived in a nondiscriminating school district.*

Proof. White plantation owner can choose to hire either teachers for the children of black sharecroppers from the black population, or from the white population. Consider the case where they hire the best possible teachers available, $\varepsilon < \frac{1}{2}$. In this world white plantation owners hire the same quality teachers as they employ for the education of their children. Appendix A shows that the proportion of blacks that stay, θ, is given by:

$$\theta_t = c_{bt}^{\frac{1}{1+\delta\varepsilon\nu}} X_{bt}^{\frac{\delta\varepsilon\nu}{1+\delta\varepsilon\nu}} h_{bt}^{-1}(1+\delta\varepsilon\nu)(\delta\varepsilon\nu)^{\frac{-\delta\varepsilon\nu}{1+\delta\varepsilon\nu}} e^{\frac{f}{1+\delta\varepsilon\nu}} \tag{27}$$

If on the other hand white plantation owners hire blacks, they must pay these blacks a competitive wage. That is to say, we assume that blacks that are not working as sharecroppers are working in a competitive labor market. The only possible discrimination that blacks face outside of the plantation is in the provision of education. Appendix A shows that the share of blacks that stay under this hiring rule is given by:

$$\theta_t = c_{bt}^{\frac{1}{1+\delta\varepsilon\nu}} X_{bt}^{\frac{\delta\varepsilon\nu}{1+\delta\varepsilon\nu}} h_{bt}^{-1} R_t^{-\frac{\delta(1-2\varepsilon)\nu}{1+\delta\varepsilon\nu}}(1+\delta\varepsilon\nu)(\delta\varepsilon\nu)^{\frac{-\delta\varepsilon\nu}{1+\delta\varepsilon\nu}} e^{\frac{f}{1+\delta\varepsilon\nu}} \tag{28}$$

where $R_t > 1$ is the ratio of maximum human capital to black human capital. Assume that the children of the current white plantation owners choose optimally, so that θ_{t+1} is chosen in order to maximize the utility of white plantation owners in period $t+1$. Let h_{bt+1} be any arbirtrary human capital of black sharecropper children chosen by white plantation owners. In order to achieve h_{bt+1} white plantation owners either can hire the best teachers or black teachers. The cost of hiring the best teachers relative to hiring black teachers is given by:

$$\widehat{X}_{bt} = X_{bt} R_t^{-\frac{1-2\varepsilon}{\varepsilon}} < X_{bt} \tag{29}$$

Consider the case where the proportion of black sharecroppers that stay is held constant between these two different hiring scenarios. Equating (11) with (12), and substituting for $\widehat{X}_{bt}$ implies that black consumption when the best teachers are provided is equal to black consumption when black teachers are hired:

$$\widehat{c}_{bt} = c_{bt} \tag{30}$$

Thus total expenditures by white plantation owners on black sharecroppers is strictly less when they hire the best teachers, for any probability of staying as well as any human capital choice for the children of black sharecroppers. Thus it must be the case that white plantation owners choose to hire the best teachers for the education of black sharecropper children. Since the white plantation owner's children's problem is identical to the problem facing their parent's their optimal choice is to hire the best possible teachers for the education of their black sharecropper children. ■

Since a nondiscriminating white teacher is paid the same whether he or she is teaching blacks or whites, then teacher salaries would be identical on plantations and off of plantations. This is counterfactual with the data. Thus we assume that there are two types of human capital.[19]

C. PLANTATION OWNER CHOICES IF MIGRATION IS POSITIVE.

Given plantation owner preferences and the functional form for migration probability of black sharecroppers, the first order conditions determining optimal investments in white education, black consumption and black education are:

$$\begin{aligned}
\frac{1}{c_t} &= \frac{\alpha\delta\varepsilon\nu}{X_t} \\
\frac{N_{bt}\theta_t}{c_t} &= \frac{1}{c_t}\left\{(1-\alpha-\sigma)\frac{y_t}{\theta_t}\frac{\partial\theta_t}{\partial c_{bt}} - c_{bt}N_{bt}\frac{\partial\theta_t}{\partial c_{bt}}\right\} + \frac{\delta(1-\alpha-\sigma)\nu}{\theta_t}\frac{\partial\theta_t}{\partial c_{bt}} \\
\frac{y_t(1-\alpha-\sigma)}{c_t(1-s_{bt})} &= \frac{1}{c_t}\left\{(1-\alpha-\sigma)\frac{y_t}{\theta_t}\frac{\partial\theta_t}{\partial s_{bt}} - c_{bt}N_{bt}\frac{\partial\theta_t}{\partial s_{bt}}\right\} + \frac{\delta(1-\alpha-\sigma)\delta\varepsilon(2-\varepsilon)}{(1+\delta\varepsilon)s_{bt}}
\end{aligned}$$

Manipulation of the Euler equation for white education expenditures produces the following characterizations for white plantation consumption and expenditures on

[19] We thank Bill Dougan for this insight.

white education:

$$
\begin{aligned}
c_t &= \frac{1}{1+\alpha\delta\varepsilon\nu}\left\{y_t - N_{bt}\theta_t\left(c_{bt}+X_{bt}\right)\right\} \\
X_t &= \frac{\alpha\delta\varepsilon\nu}{1+\alpha\delta\varepsilon\nu}\left\{y_t - N_{bt}\theta_t\left(c_{bt}+X_{bt}\right)\right\}
\end{aligned}
$$

Given the functional form for the stay probability, and assuming that the migration probability is nonzero, $\theta_t = \frac{c_{bt}^{\frac{1}{1+\delta\varepsilon\nu}} s_{bt}^{\frac{\delta\varepsilon\nu}{1+\delta\varepsilon\nu}} \left(\overline{h}_t\right)^{-\frac{\delta(1-2\varepsilon)\nu}{1+\delta\varepsilon\nu}} (1+\delta\varepsilon\nu) e^{\frac{f}{1+\delta\varepsilon\nu}}}{(\delta\varepsilon)^{\frac{\delta\varepsilon\nu}{1+\delta\varepsilon\nu}} \left(h_{bt}^s\right)^{\frac{1-\delta(1-2\varepsilon)\nu}{1+\delta\varepsilon\nu}}}$, it is apparent that:

$$
\begin{aligned}
\frac{\partial\theta_t}{\partial c_{bt}} &= \frac{\theta_t}{1+\delta\varepsilon\nu}\frac{1}{c_{bt}} \\
\frac{\partial\theta_t}{\partial s_{bt}} &= \frac{\theta_t}{1+\delta\varepsilon\nu}\frac{\delta\varepsilon\nu}{s_{bt}}
\end{aligned}
$$

Using this result and manipulating the second and third Euler equations produces the following:

$$
\begin{aligned}
\frac{N_{bt}\theta_t c_{bt}}{1+\delta\varepsilon}\left[2+\delta\varepsilon+\frac{\delta\left(1-\alpha-\sigma\right)}{1+\alpha\delta\varepsilon}\right] &= \frac{\left(1-\alpha-\sigma\right)y_t}{1+\delta\varepsilon}\left[1+\frac{1}{1+\alpha\delta\varepsilon}\right] \\
\frac{N_{bt}\theta_t c_{bt}}{1+\delta\varepsilon}\left[\delta\varepsilon+\frac{\delta\left(1-\alpha-\sigma\right)\delta\varepsilon\left(2-\varepsilon\right)}{1+\alpha\delta\varepsilon}\right] &= \left(1-\alpha-\sigma\right)y_t\left[-\frac{s_{bt}}{1-s_{bt}}+\frac{\delta\varepsilon}{1+\delta\varepsilon}+\frac{\delta\left(1-\alpha-\sigma\right)\delta\varepsilon\left(2-\right.}{\left(1+\delta\varepsilon\right)\left(1+\alpha\delta\varepsilon\right)}\right.
\end{aligned}
$$

Taking ratios produces and rearranging produces a constant teachers share:

$$
s = \frac{M}{M-\left(1+\delta\varepsilon\right)\left(1+\alpha\delta\varepsilon\right)}
$$

where M is given by:

$$
M = \left(2+\alpha\delta\varepsilon\right)\left[\frac{\delta\varepsilon\left(1+\alpha\delta\varepsilon\right)+\delta\left(1-\alpha-\sigma\right)\delta\varepsilon\left(2-\varepsilon\right)}{\left(2+\delta\varepsilon\right)\left(1+\alpha\delta\varepsilon\right)+\delta\left(1-\alpha-\sigma\right)}\right]-\delta\left(1-\alpha-\sigma\right)\delta\varepsilon\left(2-\varepsilon\right)-\delta\varepsilon\left(1+\alpha\delta\varepsilon\right)
$$

We assume parameter configurations so that $M < 0$. Substituting out for s_{bt} produces the following:

$$
\begin{aligned}
c_{bt}^{\frac{1+\delta\varepsilon+\alpha+\sigma}{1+\delta\varepsilon}} &= Q\frac{Z_t L_t^\sigma h_t^\alpha\left[h_{bt}\left(1-s\right)\right]^{1-\alpha-\sigma}\overline{h}_t^{\frac{\delta(1-2\varepsilon)}{1+\delta\varepsilon}(\alpha+\sigma)}\left(h_{bt}^s\right)^{\frac{1-\delta(1-2\varepsilon)}{1+\delta\varepsilon}(\alpha+\sigma)}\left(\delta\varepsilon\right)^{\frac{\delta\varepsilon(\alpha+\sigma)}{1+\delta\varepsilon}}}{s^{\frac{\delta\varepsilon(\alpha+\sigma)}{1+\delta\varepsilon}}\left[N_{bt}\left(1+\delta\varepsilon\right)\right]^{\alpha+\sigma}} \\
Q &= \frac{\left(2+\delta\varepsilon\right)\left(1+\alpha\delta\varepsilon\right)+\delta\left(1-\alpha-\sigma\right)}{\left(1-\alpha-\sigma\right)\left(2+\alpha\delta\varepsilon\right)} \qquad (31)
\end{aligned}
$$

D. SOLVING FOR TEACHER HIRES AND CONSUMPTION WITH NO MIGRATION

Manipulating the Euler equation determining optimal teacher hires and substituting for white consumption produces:

$$\begin{aligned} 0 &= -\frac{Z_t L_t^{\sigma} h_t^{\alpha} h_{bt}^{1-\alpha-\sigma} s_{bt}^{1+\delta\varepsilon\nu}}{\left[N_{bt}\left(1-s_{bt}\right)\right]^{\alpha+\sigma}} + \frac{\delta\nu\left(1-\alpha-\sigma\right)\varepsilon Z_t L_t^{\sigma} h_t^{\alpha}\left[h_{bt}\left(1-s_{bt}\right)\right]^{1-\alpha-\sigma} s_{bt}^{\delta\varepsilon\nu}}{\left(1+\alpha\delta\varepsilon\right) N_{bt}^{\alpha+\sigma}} \\ &\quad +(\delta\varepsilon\nu)^{\delta\varepsilon\nu}(1+\delta\varepsilon\nu)^{-(1+\delta\varepsilon\nu)}\left(h_{bt}^{s}\right)^{\frac{1-\delta(1-2\varepsilon)\nu}{1+\delta\varepsilon\nu}}\left(\overline{h}_t\right)^{\frac{\delta(1-2\varepsilon)\nu}{1+\delta\varepsilon\nu}}\delta\varepsilon\nu\left[1-\frac{1-\alpha-\sigma}{1+\alpha\delta\varepsilon\nu}\right] \end{aligned}$$

For $s_{bt}=0$, the right hand side of the equation is positive. For $s_{bt}=1$, the right hand side is $-\infty$. The right hand side is a differentiable function of s_{bt} and is monotone decreasing in s_{bt}. Therefore there is only one solution to the equation. We solved for the critical value of s_{bt} using the bisection method.

E. BLACK YEOMAN MIGRATION PROBABILITY

If a black yeoman in a discriminatory district stays he receives the following utility

$$\ln\left[h_{bt}\left(1-\tau_{bt}\right)\right]+\delta\ln\left[Ah_{bt}\left(\frac{X_{bt}}{g_b h_T}\right)^{\varepsilon\nu}R_t^{(1-\varepsilon)\nu}\right]$$

Equating utilities of staying and leaving produces the following result:

$$\ln\left[1-\tau_{bt}\right]+\delta\varepsilon\nu\ln X_{bt}-\ln\left[\frac{1}{1+\delta\varepsilon\nu}\right]-\delta\varepsilon\nu\ln\left[\frac{\delta\varepsilon\nu}{1+\delta\varepsilon\nu}\right]-\delta\varepsilon\nu\ln h_{bt}+f=(1+\delta\varepsilon\nu)\ln\left[1-\varphi\right]$$

Simplifying produces:

$$\theta_t=\left(1-\tau_{bt}\right)^{\frac{1}{1+\delta\varepsilon\nu}} h_{bt}^{-\frac{\delta\varepsilon\nu}{1+\delta\varepsilon\nu}} X_{bt}^{\frac{\delta\varepsilon\nu}{1+\delta\varepsilon\nu}}(1+\delta\varepsilon\nu)(\delta\varepsilon\nu)^{\frac{-\delta\varepsilon\nu}{1+\delta\varepsilon\nu}} e^{\frac{f}{1+\delta\varepsilon\nu}}$$

A white yeoman only cares about the amount that he can extract from black yeoman remaining in his district. Thus for any given revenue per black yeoman, the discriminating white yeoman wishes to maximize the proportion that stays. Therefore the disciminating district chooses to hire the best teachers for black children as well.

The three Euler equations determining optimal choice of tax rates for whites and blacks and the level of spending on black children are:

$$\begin{aligned}
\frac{1}{1-\tau_t^y} &= \frac{\delta\varepsilon\nu h_t^y}{X_t^y} \\
\frac{\delta\varepsilon\nu N_{bt}^y\theta_t h_{bt}^y}{X_t^y} + \frac{\delta\varepsilon\nu}{X_t^y}\left\{N_{bt}^y h_{bt}^y \tau_{bt}^y - X_{bt}^y N_{bt}^y\right\}\frac{\partial\theta_t}{\partial\tau_{bt}^y} &= 0 \\
-\frac{\delta\varepsilon\nu N_{bt}^y\theta_t}{X_t^y} + \frac{\delta\varepsilon\nu}{X_t^y}\left\{N_{bt}^y h_{bt}^y \tau_{bt}^y - X_{bt}^y N_{bt}^y\right\}\frac{\partial\theta_t}{\partial X_{bt}^y} &= 0
\end{aligned}$$

Rearranging and taking the ratio of the second and third Euler equations produces:

$$X_{bt} = \delta\varepsilon\nu h_{bt}\left(1-\tau_{bt}\right)$$

Replacing this result into the third Euler equation and using the definition of θ_t produces:

$$\tau_{bt} = \frac{1+2\delta\varepsilon\nu}{2\left(1+\delta\varepsilon\nu\right)}$$

Finally using the budget constraint for white yeomen, and the previous two results allows us to calculate:

$$\tau_{wt} = \max\left\{0, \frac{\delta\varepsilon\nu}{1+\delta\varepsilon\nu} - \frac{N_{bt}\theta_t h_{bt}^y}{2\left(1+\delta\varepsilon\nu\right)h_{wt}^y}\right\}$$

Using these results for τ_{bt} and X_{bt} and substituting into the definition of θ_t produces:

$$\begin{aligned}
\theta_t &= \left(1-\tau_{bt}\right)^{\frac{1}{1+\delta\varepsilon\nu}} h_{bt}^{-\frac{\delta\varepsilon\nu}{1+\delta\varepsilon\nu}}\left(\delta\varepsilon\nu h_{bt}\left(1-\tau_{bt}\right)\right)^{\frac{\delta\varepsilon\nu}{1+\delta\varepsilon\nu}}\left(1+\delta\varepsilon\nu\right)(\delta\varepsilon\nu)^{\frac{-\delta\varepsilon\nu}{1+\delta\varepsilon\nu}} e^{\frac{f}{1+\delta\varepsilon\nu}} \\
&= \left(1-\tau_{bt}\right)\left(1+\delta\varepsilon\nu\right)e^{\frac{f}{1+\delta\varepsilon\nu}} \\
&= \frac{e^{\frac{f}{1+\delta\varepsilon\nu}}}{2}
\end{aligned}$$